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ONSET OF SLUGGING IN HORIZONTAL GAS–LIQUID PIPE FLOW

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Abstract—The onset of slugging in horizontal pipes has been analysed by a new approach. The relation between the appearance of waves on the stratified film, the formation of slugs and the transition to stable slug flow has been quantitatively examined. A new, necessary but not sufficient criterion for the transition to "stable" slug flow has been derived from an initial condition of slug growth. It reflects the well-known fact that waves of all kinds may form, even leading to liquid bridging of the pipe, but that the resulting slugs decay over a wide range of flow rates. In general, for slug flow to develop, an unstable film flow situation with waves is necessary, in addition to a condition allowing slugs, once formed, to grow initially. It is further shown, by comparisons with large-scale, high-pressure data from the SINTEF Two-phase Flow Laboratory, that this criterion for slug growth is the more restrictive, and may in practice be applied alone for the transitions from stratified or annular flow to slug flow, or even to dispersed bubble flow for high-pressure systems.

Key Words: waves, instabilities, onset of slugging

1. INTRODUCTION

Theoretical prediction of the onset of slugging in near-horizontal pipe flow has received much attention over the last few decades. Starting with Kordyban & Ranov (1970) it has been thought that the transition from stratified to slug flow could be described in terms of classical linear stability analysis. The obtained Kelvin–Helmholz (K–H) transition criterion for near-horizontal, inviscid pipe flow is usually expressed as

$$F = \frac{U_{sG}}{\sqrt{gD\sin\theta}}\sqrt{\frac{\rho_G}{\rho_L - \rho_G}} \geqslant K\epsilon^{3/2}. \qquad [1]$$

However, the derived coefficient $K = 1$ is roughly a factor of 2 too high compared with experimental data, for instance those of Wallis & Dobson (1973). Much of the later work applying K–H theory has been focused on explaining this discrepancy. Taitel & Dukler (1976) argued that

$$K = \left(1 - \frac{h}{D}\right)\sqrt{\frac{A}{D\dfrac{dA_L}{dh}}}, \qquad [2]$$

based on viscous theory, although criterion [1] was derived assuming inviscid flow; D is the pipe diameter θ is the inclination to the gravity vector, U_{sG} is the superficial gas velocity, ρ is the density, ϵ is the gas fraction, A is the pipe cross-section and h is the height of the liquid film, as indicated in figure 1.

Mishima & Ishii (1980) extended the analysis of Kordyban & Ranov (1970). Using the concept of the fastest growing wave they obtained the necessary factor $K = 1/2$ in [1] theoretically.

However, as pointed out by Lin & Hanratty (1986), among others, inviscid K–H instability theory implies that liquid inertia does not contribute to the instability; the forces causing the instability are in phase with the wave height, and viscous shear stress terms are unimportant. Another, less noticed defect of all K–H instability theories is that at neutral stability the wave speed $C = U_L$ (if $\rho_G \ll \rho_L$), which is never observed. Lin & Hanratty (1986) extended their linear stability analysis to include inertia and viscous effects, and derived a theoretical expression for K yielding good agreement with reported low-pressure, small-scale data for low and medium gas flow rates, before wave coalescence effects were thought to be dominating.

In this paper the effect of applying different stratified mean flow models to the transition has been examined by an extension of the linear theory of Lin & Hanratty (1986). Several types of interfacial friction factors were investigated. Their effect on the transition to wavy flow is found to be considerable.

More interesting, recent approaches by Ferschneider *et al.* (1985), Wu *et al.* (1987), Watson (1989) and Barnea & Taitel (1989) consider the complete non-linear dynamic one-dimensional equations. Watson (1989) showed that non-linear roll waves are possible solutions of the one-dimensional mass and momentum equations for gas and liquid. Periodic waves may be constructed by fitting together piecewise continuous solutions by shocks or hydraulic jumps, as suggested by Dressler (1949). Watson (1989) derived an expression for the maximum height of a roll wave, and identified the transition to slug flow to occur when this wave just touched the top of the pipe. Although these criteria are in reasonable agreement with low pressure 1" i.d. pipe data, substantial discrepancies occur when comparing with large-scale high-pressure data.

This indicates that the transition to slug flow cannot be described in terms of wave formation alone, see for instance Wu *et al.* (1987) and Ruder *et al.* (1989). This leads to an interesting question: assuming that a slug has been formed, regardless of the precise mechanism, under which conditions would it grow and lead to a stable slug flow? An obvious requirement is that the slug front velocity must initially exceed the tail velocity for some period of time. Based on this, a simple criterion for the transition is derived in terms of the average parameters of the previous stratified flow condition.

The discrepancy between the onset of waves and the transition to slug flow depends on the fluid properties and pipe diameter. It increases significantly with increasing pressure. In air–water atmospheric pressure 1" i.d. pipe experiments, the condition for slug growth is satisfied at a lower liquid flow rate than that giving the onset of roll waves, over a wide range of gas flow rates. Thus, the transition to roll waves (touching the top of the pipe) would be expected to coincide with the transition to slug flow, as observed. Comparisons with high pressure (20 and 30 bar) diesel–nitrogen large-scale data from the SINTEF Two-phase Flow Laboratory show that the situation is reversed. The liquid flow rate must be increased by a factor of 2–3 to satisfy the condition for slug growth, which now coincides with the transition to stable slug flow.

2. ONSET OF SLUGGING

2.1. Smooth-to-wavy stratified flow transition

Consider the one-dimensional transient two-fluid model for stratified flow. The flow, depicted in figure 1, is assumed to be co-current gas–liquid in a pipe of diameter D, or a channel of height H and infinite width, with an inclination θ to the gravity vector. Neglecting surface tension, the conservation equations for liquid mass and momentum are expressed as follows:

$$\frac{\partial A_L}{\partial t} + \frac{\partial}{\partial x}[A_L U_L] = 0 \qquad [3]$$

and

$$\frac{\partial}{\partial t}[A_L U_L] + \frac{\partial}{\partial x}[A_L \Gamma_L U_L^2] = -\frac{A_L}{\rho_L}\left[\frac{\partial P_i}{\partial x} + \rho_L g \sin\theta \frac{\partial h}{\partial x}\right] + \frac{1}{\rho_L}(\tau_i S_i - \tau_L S_L) + g A_L \cos\theta, \qquad [4]$$

where A_L and A_G are the cross-sectional areas occupied by the liquid and gas phases, respectively. U_G and U_L are the local gas and liquid velocities and Γ_k is a shape factor, defined by

$$\Gamma_k = \frac{1}{A_k U_k^2}\int_{A_k} U^2 \, dA. \qquad [5]$$

Other geometrical parameters are defined in figure 1. The shallow water approximation has been applied to obtain the local liquid pressure as

$$p = P_i + \rho_L g(h - y)\sin\theta, \qquad [6]$$

where h is the local liquid height. Similarly, the gas phase conservation of mass and momentum may be expressed as

$$\frac{\partial A_G}{\partial t} + \frac{\partial}{\partial x}[A_G U_G] = 0 \qquad [7]$$

and

$$\frac{\partial}{\partial t}[A_G U_G] + \frac{\partial}{\partial x}[A_G \Gamma_G U_G^2] = -\frac{A_G}{\rho_G}\left[\frac{\partial P_i}{\partial x} + \rho_G g \sin\theta \frac{\partial h}{\partial x}\right] - \frac{1}{\rho_G}(\tau_G S_G + \tau_i S_i) + g A_G \cos\theta. \quad [8]$$

Lin & Hanratty (1986) performed a linear stability analysis on the set of equations [3]–[8], assuming that the gas and liquid flows may be described as a mean flow ($\bar{U}_G$, $\bar{U}_L$, $\bar{h}, \ldots$) and a perturbation of the form (U'_G, U'_L, h' etc.), e.g.

$$h = \bar{h} + h', \quad [9]$$

where

$$h' = \hat{h}\exp[ik(x - Ct)].$$

By introducing dimensionless quantities (overhead $\sim$) with respect to the pipe diameter D, other geometrical parameters in figure 1 may be defined as follows:

$$\tilde{h} = \frac{h}{D}, \qquad \gamma = 2\cos^{-1}(1 - 2\tilde{h}),$$

$$\tilde{S}_i = 2\cdot\sqrt{\tilde{h} - (\tilde{h})^2}, \qquad \tilde{S}_L = \cos^{-1}(1 - 2\tilde{h}),$$

$$\tilde{S}_G = \pi - \tilde{S}_L, \qquad \tilde{A}_L = \tfrac{1}{4}[\tilde{S}_L - \tilde{S}_i(1 - 2\tilde{h})].$$

$$\tilde{A}_G = \frac{\pi}{4} - \tilde{A}_L, \quad [10]$$

Each geometrical quantity is expressed as the sum of an average and a fluctuating component. The real part of the wave velocity (C_R) is then obtained from the imaginary parts of the neutral stability equations. C_R is a complicated function of shear stresses and wetted perimeters, as shown by Espedal & Bendiksen (1989).

The resulting condition for neutral stability in pipe flow, obtained by Lin & Hanratty (1986) was given as

$$\left(\frac{C_R}{\bar{U}_L} - 1\right)^2 \frac{1}{gD\sin\theta}\left(\frac{\bar{\tilde{A}}^2}{\bar{\tilde{A}}_L^3}\right)\left(\frac{\hat{\tilde{A}}_L}{\hat{\tilde{h}}}\right)U_{sL}^2 + \frac{\rho_G}{\rho_L}\left(\frac{\bar{\tilde{A}}^2}{\bar{\tilde{A}}_G^3}\right)\left(\frac{\hat{\tilde{A}}_L}{\hat{\tilde{h}}}\right)\frac{1}{gD\sin\theta}U_{sG}^2 - 1 = 0. \quad [11]$$

Performing essentially the same stability analysis, Espedal & Bendiksen (1989) obtained a slightly different criterion:

$$\left[\left(\frac{C_R}{\bar{U}_L} - 1\right)^2 \frac{1}{gD\sin\theta}\left(\frac{\bar{\tilde{A}}^2}{\bar{\tilde{A}}_L^3}\right)\frac{\hat{\tilde{A}}_L}{\hat{\tilde{h}}} + \frac{\rho_G}{\rho_L}\frac{1}{gD\sin\theta}\frac{\hat{\tilde{A}}_L}{\hat{\tilde{h}}}\left(\frac{\bar{\tilde{A}}^2}{\bar{\tilde{A}}_L^2\bar{\tilde{A}}_G}\right)\left(\frac{C_R}{\bar{U}_L}\right)^2\right]U_{sL}^2$$
$$- \frac{\rho_G}{\rho_L}\frac{2}{gD\sin\theta}\frac{\hat{\tilde{A}}_L}{\hat{\tilde{h}}}\left(\frac{\bar{\tilde{A}}^2}{\bar{\tilde{A}}_G^2\bar{\tilde{A}}_L}\right)\left(\frac{C_R}{\bar{U}_L}\right)U_{sG}U_{sL}$$
$$+ \frac{\rho_G}{\rho_L}\frac{1}{gD\sin\theta}\frac{\hat{\tilde{A}}_L}{\hat{\tilde{h}}}\left(\frac{\bar{\tilde{A}}^2}{\bar{\tilde{A}}_G^3}\right)U_{sG}^2 - \frac{(\rho_L - \rho_G)}{\rho_L} = 0. \quad [12]$$

Comparison of [12] and [11] shows that in the analysis of Lin & Hanratty (1986), terms which are of the same order as the second term in [11] have been neglected. These terms are not significant at low pressures (due to the density ratio ρ_G/ρ_L). More importantly, expression [12] enables a significant simplification, observing that

$$\frac{\tilde{A}}{\bar{\tilde{A}}_L} = \frac{1}{1 - \epsilon}, \quad [13]$$

$$\frac{\tilde{A}}{\bar{\tilde{A}}_G} = \frac{1}{\epsilon} \quad [14]$$

and

$$\frac{\hat{\hat{A}}_{\mathrm{L}}}{\tilde{\tilde{h}}}=\frac{\mathrm{d}A_{\mathrm{L}}}{\mathrm{D}\mathrm{d}h}, \tag{15}$$

where $\mathrm{d}A_{\mathrm{L}}/\mathrm{d}h = \mathrm{D}\sin\gamma/2 = S_{\mathrm{i}}$.

Condition [12] may then be reformulated as

$$(C_{\mathrm{R}}-\bar{U}_{\mathrm{L}})^2+\frac{\rho_{\mathrm{G}}}{\rho_{\mathrm{L}}}\frac{1-\epsilon}{\epsilon}(C_{\mathrm{R}}-\bar{U}_{\mathrm{G}})^2=\frac{\rho_{\mathrm{L}}-\rho_{\mathrm{G}}\cdot gA_{\mathrm{L}}\sin\theta}{\rho_{\mathrm{L}}\dfrac{\mathrm{d}A_{\mathrm{L}}}{\mathrm{d}h}}. \tag{16}$$

It is interesting to observe that [16] reduces to the standard K–H criterion [1] for $C_{\mathrm{R}} = \bar{U}_{\mathrm{L}}$. The onset of liquid film instabilities, as predicted by either linear (Espedal & Bendiksen 1989; Lin & Hanratty 1986) or non-linear theory, may then, in general, be applied to describe the transition from smooth to wavy stratified flow. As stated in the Introduction, a frequent, additional (implicit) assumption, made by Taitel & Dukler (1976), Lin & Hanratty (1986), Ferschneider *et al.* (1985) and Watson (1989), among others, has been that these instability criteria also give the transiton boundary to slug flow. That is, they provide necessary as well as sufficient conditions for the onset of slugging. As will be shown below, this is not always true.

2.2. *Conditions for slug growth*

Assume that a stratified wavy flow condition exists, and that a liquid bridging of the pipe occurs at a given point in time. If stable slug flow is to result, the slug must grow initially, as indicated in figure 2(a, b). A similar approach has been followed previously by Taitel (1987) and Asheim (1987), among others. A condition for the slug to grow is obviously that its front velocity, U_{F}, exceeds its tail velocity. The slug tail velocity equals the large Taylor bubble velocity U_{B}, which according to Bendiksen (1984) can be expressed in terms of the average liquid velocity in the slug, U_{Ls}, and a gravity-induced drift velocity, U_0:

$$U_{\mathrm{B}} = C_0 U_{\mathrm{Ls}} + U_0. \tag{17}$$

For no slip in the slug ($U_{\mathrm{Gs}} = U_{\mathrm{Ls}}$ in [20], below) this reduced to

$$U_{\mathrm{B}} = C_0 U_{\mathrm{M}} + U_0, \tag{18}$$

where U_{M} is the total superficial velocity.

The slip parameter C_0 and drift velocity U_0 are dependent on the pipe diameter and inclination, as well as a Froude number, $\mathrm{Fr} = U_{\mathrm{Ls}}/\sqrt{gD}$. For slugs of lengths $L_{\mathrm{s}} \geqslant 10\,\mathrm{D}$, the values of Bendiksen (1984) may be applied:

$$C_0=\begin{cases}1.05+0.15\cos^2\theta & \text{for Fr} < 3.5\\ 1.20 & \text{for Fr} \geqslant 3.5\end{cases}$$

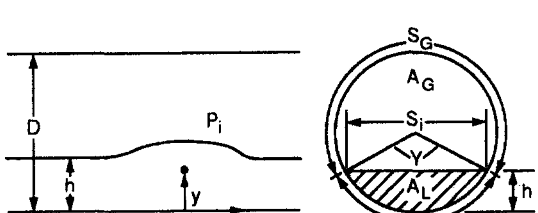

Figure 1. Definition of the geometrical parameters for idealized stratified pipe flow.

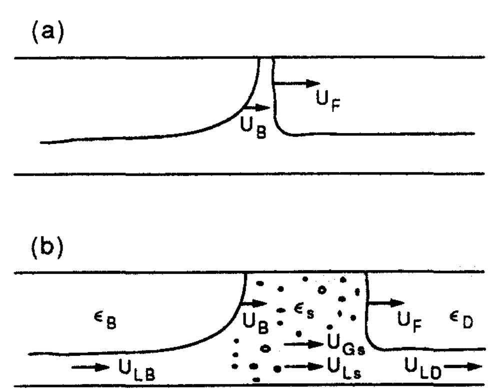

Figure 2. Initial growth of a "pseudo-slug" (b) after liquid bridging of the pipe (a).

and

$$U_0 = \begin{cases} U_{0v}\cos\theta + U_{0h}\sin\theta & \text{for Fr} < 3.5 \\ U_{0v}\cos\theta & \text{for Fr} \geqslant 3.5, \end{cases} \quad [19]$$

where

$$U_{0v} = 0.35\sqrt{gD}$$

and

$$U_{0h} = 0.54\sqrt{gD}.$$

U_{0v} and U_{0h} are the bubble velocities in stagnant liquid, neglecting surface tension, in vertical and horizontal pipes, respectively.

The validity of [17]–[19] for fully-developed slug flow is now well-established over a wide range of pipe diameters, fluid properties and pressures, see for instance Brandt & Fuchs (1989). A discussion on the sensitivity of predicted transitions on the applied bubble velocity follows in section 3.2.

The liquid velocity in the slug may be obtained from a total volumetric balance:

$$U_M = U_{sL} + U_{sG} = \epsilon_s U_{Gs} + (1 - \epsilon_s) U_{Ls}. \quad [20]$$

U_{Gs} and ϵ_s are the average gas velocity and volumetric gas fraction in the slug, respectively. This gives

$$U_{Ls} = \frac{(U_M - \epsilon_s U_{Gs})}{(1 - \epsilon_s)}. \quad [21]$$

The slug front velocity U_F may be estimated from a liquid flow balance across the slug front:

$$(U_{Ls} - U_F)(1 - \epsilon_s) = (U_{LD} - U_F)(1 - \epsilon_D), \quad [22]$$

where U_{LD} and ϵ_D are the downstream liquid film velocity and gas fraction [see figure 2(a, b)]. This yields

$$U_F = \frac{U_{Ls}(1 - \epsilon_s) - U_{LD}(1 - \epsilon_D)}{\epsilon_D - \epsilon_s}. \quad [23]$$

In terms of the gas and liquid superficial velocities, U_{sGD} iand U_{sLD}, in the downstream stratified flow:

$$U_F = \frac{U_{Ls}(1 - \epsilon_s) - U_{sLD}}{\epsilon_D - \epsilon_s}, \quad [24]$$

which, using [21], reduces to

$$U_F = \frac{U_M - U_{Gs}\epsilon_s - U_{sLD}}{\epsilon_D - \epsilon_s}. \quad [25]$$

In a steady-state stratified flow condition, prior to slug growth, $U_M = U_{sLD} + U_{sGD}$, which finally gives

$$U_F = U_{GD}\frac{\epsilon_D - \epsilon_s \dfrac{U_{Gs}}{U_{GD}}}{\epsilon_D - \epsilon_s}. \quad [26]$$

In the general case there may be droplets present in the gas, and [22] is modified to

$$(U_{Ls} - U_F)(1 - \epsilon_s) = (U_{LD} - U_F)\beta_D + (U_{dD} - U_F)\gamma_D, \quad [27]$$

where β_D and γ_D are the downstream liquid film and droplet fractions, respectively. This gives

$$U_F = \frac{U_{Ls}(1 - \epsilon_s) - \beta_D U_{LD} - \gamma_D U_{dD}}{\epsilon_D - \epsilon_s}, \quad [28]$$

which reduces to [24]. It might seem surprising that the front velocity formulas are identical, with or without droplets, but the velocities generally are very different. For instance the gas velocity, U_{GD}, will be much lower in the case of droplet flow, as will the downstream liquid film fraction, β_D.

The criterion for sustaining slug flow now becomes

$$U_B < U_F \qquad [29]$$

or

$$U_B < U_{GD} \frac{\epsilon_D - \epsilon_s \dfrac{U_{Gs}}{U_{GD}}}{\epsilon_D - \epsilon_s}. \qquad [30]$$

Condition [30] contains no information on whether a slug actually forms under given flow conditions. To determine this, one of the criteria in section 2.1 should be applied. Criterion [30], however, gives the lower liquid flow limit for sustaining stable slug flow. As will be shown, with the exception of low-pressure systems, this is the more restrictive condition.

The criterion [30] also explains quantitatively the observed strong pressure effect on the transition from stratified to slug flow, and why the slug region shrinks with increasing pressure. Assuming, for simplicity, no liquid droplets, increasing the system pressure, keeping the individual gas and liquid flow rates contstant, leads to a decrease in liquid holdup in stratified flow due to increased gas wall and interfacial friction. For low and moderate gas rates, where $\epsilon_s = 0$, U_B is independent of pressure, whereas U_{GD} decreases with increasing pressure (as the void increases). According to [30] it then becomes more difficult for the slug to grow, once formed, thus shifting the transition towards higher superficial liquid velocities.

An important special case occurs when $\epsilon_s = 0$. Relation [30] then simply reduces to

$$U_B < U_{GD}. \qquad [31]$$

U_{GD} is the gas velocity in stratified flow prior to the transition to slug flow, as the downstream conditions are not significantly changed by the appearance of a single slug in the line. An interesting observation is that using [21] and [22], and $C_0 = 1.0$ ($\mathrm{Fr} < 3.5$ in [19]), condition [31] becomes identical to relation [38] of Ruder *et al.* (1989), neglecting void in the slugs.

Relation [31] has been known to apply empirically to the transition from annular to slug flow, as reported by Wallis (1969), and forms the basis for the dispersed–separated flow criterion applied in the OLGA model. In fact, it can be shown, see the Appendix, that criterion [30] is mathematically equivalent to the "minimum slip" criterion applied to the OLGA model.

It is further interesting to observe that [30] not only provides a necessary condition for stable slug flow in the transition from stratified to slug flow. It equally well applies to annular flow, but with U_{GD} and ϵ_D now being the gas velocity and void fraction corresponding to the initial annular flow condition. Insofar as the transition from slug to dispersed bubble flow may be interpreted as a limiting form of slug flow, with the slug fraction approaching 1, [30] then applies to the transition from annular to dispersed bubble flow as well.

Thus, [30] becomes a very general necessary condition for the transition from separated (stratified or annular) to dispersed (slug or bubble) flow.

3. RESULTS AND DISCUSSION

3.1. Comparisons with small-scale low-pressure data

Verification of our linear stability analysis was made against the results of Lin & Hanratty (1986) for neutral stability in pipe flow. The predictions were also compared with Lin's (1984) air–water experimental data at 1 bar in 2.54 and 9.53 cm i.d. pipes, as shown in figure 3. The results scale very well with the defined Fr number, and using an interfacial friction factor twice that of stratified smooth flow, the agreement with the data is also satisfactory. Applying the smooth flow value yields a transition line close to that of Taitel & Dukler (1976).

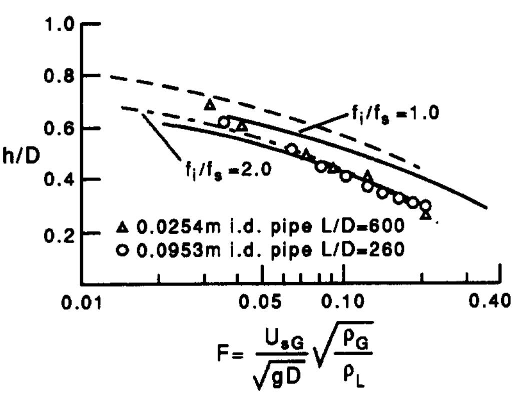

Figure 3. The linear stability criterion ([12], ——), compared with those of Lin & Hantratty (1986) (– – – –) and Taitel and Dukler (1976) (– – –) for fully-developed horizontal pipe flow. Atmospheric air–water data of Lin (1985) are also included (△, $D = 2.54$ cm; ○, $D = 9.53$ cm; $\rho_G/\rho_L = 1.12$, $\nu_G/\nu_L = 16.1$).

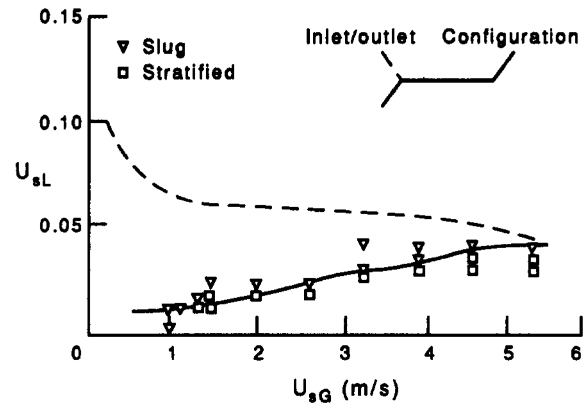

Figure 4. Experimentally observed transitions from stratified to slug flow in atmospheric horizontal air–water pipe flow (Bendiksen & Malnes 1987), starting with slug flow at the inlet compared with the slug growth criterion ([30], ——). The observed transition with stratified smooth flow at the inlet (– – –) is included as a reference.

Condition [30] for slug growth is satisfied for a significantly lower liquid flow rate. This implies that "proto" slugs, once formed will continue to grow, and the criterion for onset of waves also determines the onset of stable slugging in these air–water low-pressure systems. To verify this hypothesis, we have compared [30] with the data of Bendiksen & Malnes (1987) in figure 4. In these experiments the effects of different inlet and outlet conditions on the transition from stratified to slug flow were investigated in horizontal air–water pipe flow ($D = 2.4$ cm). Starting with slug flow at the inlet of the test section, stable slug flow is obtained at much lower liquid flow rates than with stratified smooth flow at the inlet. The experimentally observed lower liquid flow limit for sustaining slug flow imposed at the inlet (see figure 4) coincides very well with the predicted slug growth limit from [30]. Relation [30] thus constitutes a necessary condition for slug flow.

3.2. Comparisons with the SINTEF high-pressure data

The criterion for slug growth [30] using the original OLGAS mean flow model, has been compared with large-scale (18.9 cm i.d. L/D ratio of 2000) high-pressure data from the SINTEF Two-phase Flow Laboratory, see Brandt & Fuchs (1989) and Bendiksen *et al.* (1986) for a detailed description of the experimental conditions. Predictions and observed slug flow transitions are shown in figures 5 and 6, for horizontal, 20 and 30 bar, nitrogen–diesel data.

The linear stability criterion obtained in this paper, [12], has also been compared with the 30 bar data in figure 6. Three different mean flow models were applied; that of Lin & Hanratty (1986), and two based on the point model OLGAS (Bendiksen *et al.* 1988, 1991), using the interfacial friction factors of Wallis (1969) and Moody (with zero roughness). To enable a meaningful comparison, the droplet field in OLGAS was turned off.

As can be seen from figure 6, the complete stability criterion from this paper, [12], using either the model of Lin & Hanratty (1986) or OLGAS for the mean flow, gives transition to slug flow at too low U_{sL} for 30 bar. The data show a slightly increasing critical U_{sL} for higher U_{sG}. The slug growth criterion [30] reproduces this tendency, but the criteria [11] and [12] do not. The Lin & Hanratty (1986) criterion, [11], using either their mean flow model or OLGAS with the interfacial friction factor of Wallis (1969), breaks down for superficial gas velocities higher than approx. 2 m/s. Results are also presented using OLGAS for the mean flow with Moody's friction factor with zero roughness at the gas–liquid interface. In this case, however, the ratio between the interfacial and the gas wall friction factor becomes less than one, which is unphysical.

Figure 7 shows $C_R/\bar{U}_L$ and C_R used in [12] for OLGAS and Lin & Hanratty's (1986) mean flow models, respectively. The wave velocity is quite constant (slightly above 1 m/s) for the superficial gas velocities applied. The ratio between C_R and $\bar{U}_L$ is high for low U_{sG}.

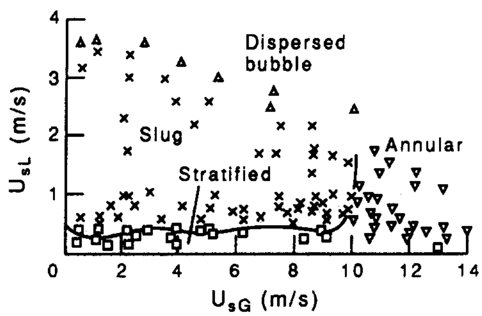

Figure 5. Observed flow regime transitions from stratified to slug flow in horizontal diesel–nitrogen pipe flow at 20 bar from the SINTEF Two-phase Flow Laboratory [Brandt & Fuchs (1989); □, stratified flow, ×, slug flow]. The predicted transition line is from the slug growth criterion ([30], ——).

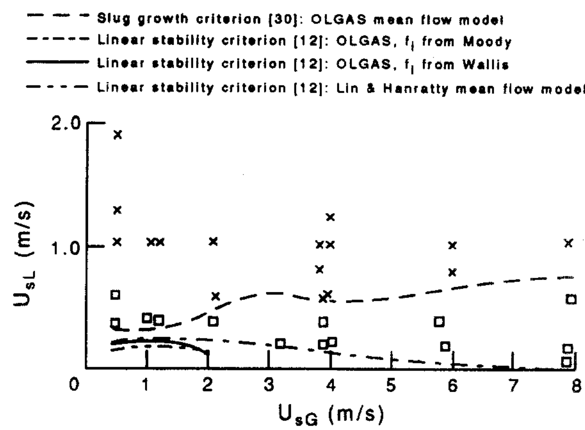

Figure 6. Observed flow regime transitions from stratified to slug flow in horizontal diesel–nitrogen pipe flow at 30 bar from the SINTEF Two-phase Flow Laboratory [Bendiksen *et al.* (1986); □, stratified flow, × slug flow]. The predicted transition lines are from the slug growth criterion [30] and linear stability theory, applying different mean flow models.

Finally, figure 8 shows a sensitivity study on the parameters of the slug formula [19] for diesel–nitrogen flow in a horizontal pipe of 19 cm i.d. at 90 bar pressure. Over most of the transition region $C_0 = 1.05$, from [19]. A variation in C_0 by a factor from 0.95 to 1.10, as shown in figure 8, corresponds to C_0 being changed from 0.99 to 1.16. The largest relative change in the predicted transition is at about $U_{sG} = 2.2$ m/s, where it is +20%. This is still qualitatively correct, and should be compared with the Lin & Hanratty (1976) criterion for neutral stability in pipe flow [11] which gives a slug transition an order of magnitude too low. The influence of the extra terms, derived in this paper [12], shifts the transition to even lower values of U_{sL}.

Thus, at high pressures there is a large region where waves or "proto-slugs" may exist, without giving stable slug flow. The onset of slugging is then very well described by the criterion [30] for slug growth.

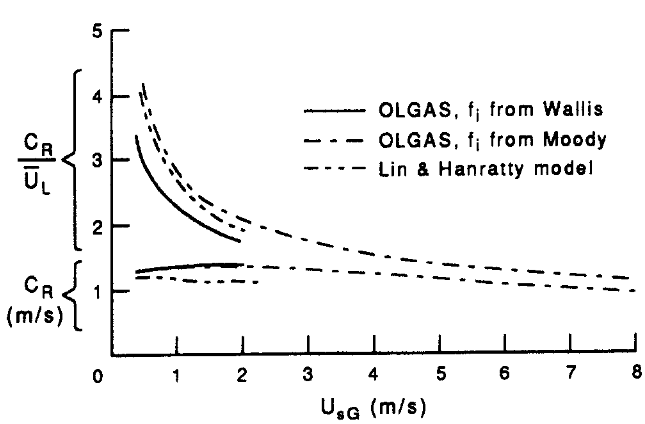

Figure 7. Predicted wave velocity at the transition from stratified to slug flow, applying the complete stability criterion [12], and different mean flow models (horizontal diesel–nitrogen flow at 30 bar in the SINTEF Two-phase Flow Laboratory.)

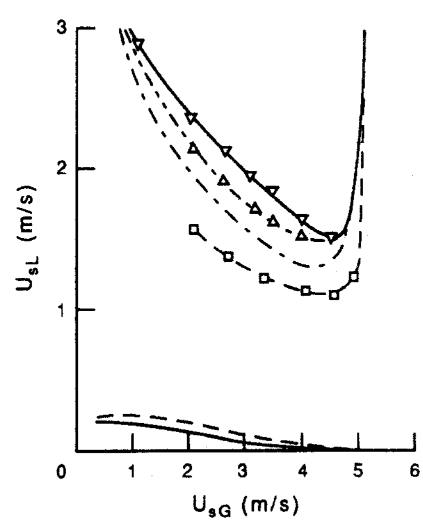

Figure 8. The sensitivity of the slug bubble velocity [17] on the predicted transition from stratified to slug flow, using the slug growth criterion [30] (———), C_0 from [19]; □, $C_0^* = 0.95\,C_0$; △, $C_0^* = 1.05\,C_0$; ▽, $C_0^* = 1.10\,C_0$). The linear stability criterion ([12], ——) and that of Lin & Hanratty ([11], ——) are included for reference. Horizontal diesel–nitrogen pipe flow (i.d. = 19 cm) at 90 bar.

4. CONCLUSIONS

A general linear instability model has been established, accepting different closure laws for the stratified mean flow. In particular, the effect of different interfacial shear stress models on the instability has been investigated.

Predictions of the onset of instabilities and slugging have been compared with large-scale horizontal nitrogen–diesel oil data at 20 and 30 bar with an i.d. of 19 cm and an L/D of 2000 from the SINTEF Two-phase Flow Laboratory. The results show that the onset of waves depends on the stratified mean flow model applied. The effect of using the complete linear stability criterion [12] is significant at higher pressures, only.

More importantly, the relation between the onset of waves, liquid bridging of the pipe and the onset of stable slug flow is clarified. A new practical engineering criterion for slug growth is presented, providing a necessary but not sufficient condition for the onset of stable slug flow. At low pressures, condition [30] for slug growth is satisfied for a significantly lower liquid flow rate than condition [12] giving the onset of waves. Consequently, for these systems, within a large range of flow rates, stratified flow is conditionally stable, only. Imposing (external) perturbations on the flow, e.g. as done in a series of experiments by Bendiksen & Malnes (1987), would then lead to slug flow in this region.

For high-pressure large-diameter pipes the situation becomes reversed. The liquid flow rate must be increased by a factor of 2–3 above that of [12] to satisfy the condition for slug growth [30], which now coincides with the transition to stable slug flow. Waves appear more easily on the film with increasing pressure, but due to the increased gas friction and thus lower liquid holdup, it becomes more difficult for these waves and proto-slugs to grow and form stable slug flow, as reflected by criterion [30].

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APPENDIX

Alternative Form of the Slug Growth Criterion

Assuming uniform slug flow, as in the OLGA model, the flow is idealized to consist of identical slug units of length (L). The average void fraction, which equals that of each slug unit, is then given as

$$\epsilon = \frac{L_s}{L}\epsilon_s + \frac{L_B}{L}\epsilon_B \qquad [A.1]$$

or

$$\epsilon = \epsilon_s + \frac{L_B}{L}(\epsilon_B - \epsilon_s). \qquad [A.2]$$

An expression for the slug bubble fraction may be obtained from an average of the superficial gas velocity over the time of passage of each slug unit, as shown by Malnes (1983):

$$\frac{L_B}{L} = \frac{U_{sG} - \epsilon_s U_{Gs}}{U_B(\epsilon_B - \epsilon_s)}. \qquad [A.3]$$

Then [A.2] modifies to

$$\epsilon = \epsilon_s + \frac{U_{sG} - \epsilon_s U_{Gs}}{U_B}. \qquad [A.4]$$

Solving for U_B gives

$$U_B = \frac{U_{sG} - \epsilon_s U_{Gs}}{\epsilon - \epsilon_s}. \qquad [A.5]$$

The slug growth criterion [30] in the form

$$U_B < \frac{U_{sG} - \epsilon U_{Gs}}{\epsilon_D - \epsilon_s} \quad [A.6]$$

may then be modified, appling [A.5], as

$$U_B = \frac{U_{sG} - \epsilon_s U_{Gs}}{\epsilon - \epsilon_s} < \frac{U_{sG} - \epsilon_s U_{Gs}}{\epsilon_D - \epsilon_s}. \quad [A.7]$$

Since all factors are positive, this reduces to

$$\epsilon > \epsilon_D \quad [A.8]$$

or, in terms of the average gas velocities U_{Gav} and U_{GD} in slug and stratified flow, respectively, simply to

$$U_{Gav} = \frac{U_{sG}}{\epsilon} < \frac{U_{sG}}{\epsilon_D} = U_{GD}. \quad [A.9]$$

That is, the minimum slip criterion applied in the OLGA model is identical to the slug growth condition [30].